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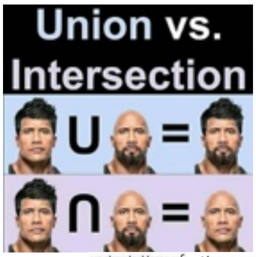
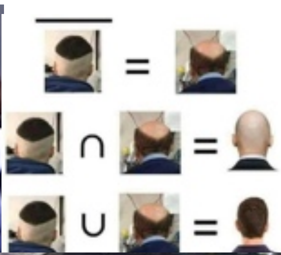
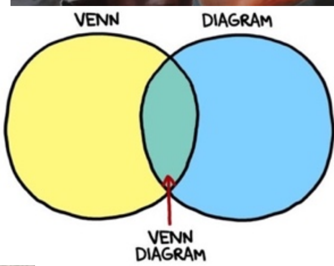
Probability 1: Venn Diagrams







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This is a long worksheet to cater for students that want extra practice. If you want a shortcut, but still be sure to cover one of each type then follow the pink highlighted questions.

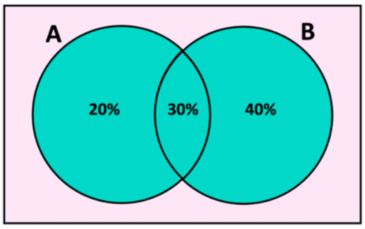
1 Bronze

1.1 Without Set Notation

1) In a group of 100 people, 40 own a cat, 25 own a dog and 15 own a cat and a dog. Find the probability that a person chosen at random:
   i. Owns a dog or cat
   ii. Owns a dog or cat, but not both
   iii. Owns a dog, given that he owns a cat
   iv. Does not own a cat, given that he owns a dog

2) In a class of 20 students, 12 study biology, 15 study history and 2 students study neither Biology nor History.
   i. Find the probability that a randomly selected student from this class is studying both Biology and History
   ii. Given that a randomly selected student studies Biology, find the probability that this student also studies History

3) Use the probability distribution of events A and B, as shown in the Venn diagram, to answer the following questions:



   i. What is the probability of event A occurring?
   ii. What is the probability of events A and B both occurring?
   iii. What is the probability of events A or B occurring?
   iv. What is the probability of event B occurring, but not A?
   v. What is the probability of neither event A nor event B occurring?
   vi. What is the probability of event B occurring?
   vii. What is the probability of event B occurring, giving that event A has occurred?

4) A café serves sandwiches and cake. Each customer will choose one of the following 3 options; buy only a sandwich, buy only a cake, or buy both a sandwich and a cake. The probability that a customer buys a sandwich is 0.72 and the probability that a customer buys a cake is 0.45. On a typical day 200 customers come to the café. Find the probability that a customer chosen at random will buy:
   i. Both a sandwich and a cake
   ii. Only a sandwich
   iii. Find the expected number of cakes sold on a typical day

5) Celeste wishes to hire a taxicab from a company, which has a large number of taxicabs. The taxicabs are randomly assigned by the company.
The probability that the taxicab is yellow is 0.4
The probability that the taxicab is a Fiat is 0.3
The probability that a taxicab is a yellow or a fiat is 0.6

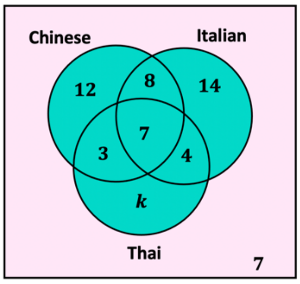
Find the probability that the taxicab hired by Celeste is **not** a yellow fiat

6) Researchers are interested in the relationship between cigarette smoking and lung cancer. Suppose an adult male is randomly selected from a particular population. The following table shows some probabilities involving the compound event that the individual does or does not smoke and the person is or is not diagnosed with cancer. Fill in the missing probability below for those who do not smoke and do not get cancer. Then answer the questions which follow. Round your answers to the nearest percent.

Smokes and gets cancer = 0.05	0.05
Smoke and does not get cancer = 0.20	0.20
Does not smoke and gets cancer= 0.03	0.03
Does not smoke and does not get cancer	?

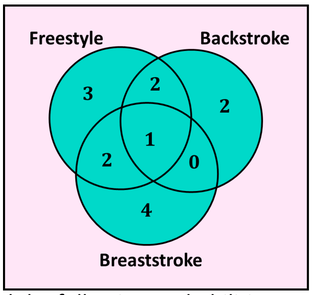
i. Find the probability that the individual gets cancer, given that he is a smoker
ii. Find the probability that the individual does not get cancer, given that he is a smoker
iii. Find the probability that the individual gets cancer, given that he does not smoke
iv. Find the probability that the individual does not get cancer, given that he does not smoke

7) There are 60 members of a club. The members indicate their liking for Chinese, Italian and Thai takeaway food in the Venn diagram below. If a member is selected at random, what is the probability that they like:



i. Italian
ii. Only Italian
iii. None of these choices
iv. All of the choices
v. Only two types of these choice
vi. Thai or Italian
vii. Italian & Thai
viii. At least one of these choices
ix. Chinese or Italian, but not Thai
x. Chinese and Italian, but not Thai
xi. Exactly one of the three choices of takeaway

8) A swim team has 25 members, and the Venn diagram shows the events which some of the swimmer participate.



If a swimmer is selected at random, find the following probabilities:

i. P(swims only freestyle)
ii. P(swims exactly two events)
iii. P(swims backstroke or breaststroke)
iv. P(swims freestyle and backstroke)
v. P(swims backstroke given swims freestyle)
vi. P(does not swim freestyle or backstroke or breaststroke)

1.2 With Set Notation

9) Shade the following
   i. P(A)
   ii. P(B)
   iii. P(A’)
   iv. P(B’)

10) Shade the following
   i. P(A∪B)
   ii. P(A∩B)
   iii. P(A’∩B’)
   iv. P(A’∩B)
   v. P(A∩B’)
   vi. P(A∪B’)
   vii. P(A’∪B)
   viii. P(A’∪B’)
   ix. P(A∪B)’
   x. P(A∩B)’

11) P(A)= 0.35, P(B)= 0.45, P(A∩B)= 0.13. Find P(A∪B) and $P(A \cap B')$

12) P(A)=0.2, P(B)=0.5. Find P(A∪B) when
   i. A and B are mutually exclusive
   ii. A and B are independent

13) $P(A) = 0.38, P(B) = 0.42, P(A \cup B) = 0.56$. Find
   i. $P(A \cap B)$
   ii. $P(A' \cap B)$
   iii. $P(A \cap B')$
   iv. $P(A' \cup B)$
   v. $P(A \cap B)'$

14) P(A)=$\frac{11}{36}$, P(B)=$\frac{1}{6}$, P(A ∪ B)′=$\frac{21}{36}$. Find
   i. P(A∩B)
   ii. P(A' ∩B)
   iii. P($A \cap B'$)
   iv. P(A' ∪B)

1.2.1 Independence/ Mutually Exclusive

15) P(A)=0.40, P(B)=0.55, P(A∪B)=0.7. Find
   i. P($A \cap B$)
   ii. P($A \cap B'$)

16) If events A and B are independent and P(A)=0.4 and P(B)=0.25. Find
   i. P(A∪B)
   ii. P(A∩B)
   iii. P(A∩B’)
   iv. P(A’∩B’)
   v. P($A \cup B'$)

17) If events A and B are such that they are independent and P(A)=0.3, P(B)=0.5, find:
   i. P(A∩B)
   ii. P(A∪B)
   iii. Are events A and B mutually exclusive?

1.2.2 Conditional

18) Given $P(A \cap B) = 0.12$ and $P(A) = 0.2$, find $P(B|A)$

19) P(A)= $\frac{2}{3}$, P(B)= $\frac{1}{2}$, P(A∩B)= $\frac{1}{4}$. Find P(A∪B) P(A∩B)’ and P(B|A)

20) Events C and D are such that P(C) = 0.6, P(D) = 0.3 and P(C∪D) = 0.8. Find P(D| C’)

21) If A and B are independent events and $P(A) = 0.5$ and $P(B) = 0.4$, find $P(B|A)$

2 Silver

2.1 Without Set Notation

22) In a group of 300 people surveyed
46 like only Cola A
23 like only Cola B
18 like only Cola C
80 like both Colas A and B
66 like both Colas A and C
45 like both Colas B and C
12 like all three Colas
Find the probability that a selected person
   i. Likes all three colas
   ii. Likes colas A and B
   iii. Likes Cola A or Cola B
   iv. Does not like Cola A or B or C
   v. Likes at least 2 Colas
   vi. Likes only 1 Cola
   vii. Likes B and does not like C

23)
96 people like wine A
93 people like wine B
96 people like wine C
92 people like wine A and B
91 people like wine B and C
93 people like wine A and C
90 like all three
There are 100 people in total
Find the probability that a person
   i. Doesn't like any wine
   ii. Likes wine A, but not wine B
   iii. Likes any wine except C
   iv. Likes exactly one of the wines
   v. Likes exactly two kinds of wine
   vi. Likes at least one wine
   vii. Likes wine A but not B or C
   viii. Likes A and B but not C
   ix. Likes wine A or B
   x. Likes wine A or B or both
   xi. Given that the person likes wine A, what is the probability that they like wine
   xii. Given that the person likes wine A or B or both, what is the probability that they do not like A
   xiii. Are A and B independent?

2.2 With Set Notation

2.3 Independence

24) Events A and B are independent with $P(A \cap B) = 0.2$ and $P(A' \cap B) = 0.6$
    i. Find $P(B)$
    ii. Find $P(A \cup B)$

25) P(A∩ B)=0.3, P(A∩ B')=0.3. A and B are independent. Find P(A∪B)

2.3.1.1 *Conditional*

26) P(A)= $\frac{2}{5}$, P(B)= $\frac{11}{20}$, P(A|B)= $\frac{2}{11}$. Find P(A∩B), P(A∪B). Are these events independent?

27) P(B)= $\frac{1}{2}$, P(A∪ B)= $\frac{13}{20}$, P(A|B)= $\frac{2}{5}$. Find P(A∩B), P(A), P(B|A), P(A'∩ B)

28) P(B)=$\frac{1}{2}$, P(AIB)=$\frac{2}{5}$, P(AUB)=$\frac{13}{20}$. Find P(A∩B), P(A'∩B), P(A), P(AIB'), P(BIA), and P(A'∪B)

2.3.1.2 *With Algebra*

29) P(A)= k, P(B)= $3k$, P(A∩B)=k^2 and P(A∪B)=0.5.
    i. Calculate k
    ii. Calculate P(A'∩B)

30) Let C and D be independent events with P(C)=$2k$, and P(D)=$3k^2$, where $0 < k < 0.5$.
    i. Write an expression for P($C \cap D$) in terms of k
    ii. Given that P($C \cap D$) =0.162, find k
    iii. Find P(C'ID)

31) Given that events A and B are independent and that P(A)=x and P(B)=y, find, in terms of x and y
    i. P($A \cap B$)
    ii. $P(A \cup B)$
    iii. P($A \cup B'$)

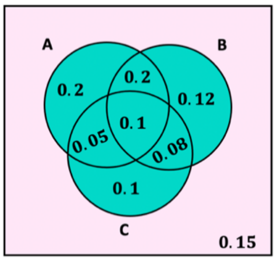
3 Gold

3.1 With Set Notation

32) Shade the following

- i. P ($(A \cup C) \cap B$)
- ii. P ($B' \cap (A \cap C)$)
- iii. P ($B' \cap (A \cup C)$)
- iv. P ($(A \cap B) \cup C$)
- v. P ($(A' \cup B') \cap C$)
- vi. $P\ ((A \cap B \cap C')')$

33) The Venn diagram shows the probabilities of 3 events, A, B and C. Find



- i. $P(A|B)$
- ii. $P(C|A')$
- iii. $P(B|A \cup B)$
- iv. $P\big((A \cap B)\big|C'\big)$
- v. $P(C|A' \cup B')$

3.1.1 Independence/Mutually Exclusive

34) A, B and C are three events with $P(A) = 0.24, P(B) = 0.4, P(C) = 0.45, P(A \cap B \cap C) = 0.1$. Given that A and B are independent, B and C are independent and $A \cap B' \cap C = \emptyset$

- i. Draw a Venn diagram to illustrate the probabilities

Find:

- ii. $P\big(A \cap (B' \cup C)\big)$
- iii. $P((A \cup B) \cap C)$
- iv. State, with reasons, whether events A' and C are independent

35) A, B and C are three events with $P(A) = 0.55, P(B) = 0.35, P(C) = 0.4, P(A \cap C) = 0.2$. Given that A and B are mutually exclusive, and B and C are independent

- i. Draw a Venn diagram to illustrate the probabilities

Find

- ii. $P(A' \cap B')$

iii. $P\big(A \cup (B \cap C')\big)$
iv. $P(A \cap C)' \cup B)$

3.1.2 With Algebra

36) $P(A' \cap B) = 0.22, P(A' \cap B') = 0.18. P(AIB) = 0.6.$
   i. Find P(A)
   ii. P(A∪B)
   iii. P(A∩B)
   iv. Are A and B independent?

37) P(A)=$\frac{1}{4}$, $P(A \cup B) = \frac{2}{3}$. A and B are independent. Find P(B), P(A'∩B) P(B'|A)

38) Consider independent events A and B. Given that P(B)=2P(A) and P(A∪B)=0.52, find P(B)

39) Events A and B are independent, and P(A∩B) =$\frac{1}{24}$ and P(A∪B) =$\frac{3}{8}$. Find P(A) and P(B)

40) Three events A, B and C are such that A and B are mutually exclusive and P(A)=0.2, P(C)=0.3 and P(A∪ $B) = 0.4$ and P(B∪ C)=0.34
   i. Calculate P(B) and P(B∩ C)
   ii. Determine whether B and C are independent

41) Two events A and B are mutually exclusive. Given that P(B)= $p \neq 0$ and P(A)=3× $P(B)$
   i. Draw a Venn diagram to illustrate the information
   ii. Find the possible values of P(B)

   Two events C and D are such that P(CID)=3 X P(C) where P(C) $\neq 0$
   iii. Explain whether or not events C and D could be independent events

   Given also that P(C∩ $D) = \frac{1}{2} \times P(C)$ and $\mathrm{P(C'} \cap D') = \frac{7}{10}$
   iv. Find $P(C)$ showing your working clearly

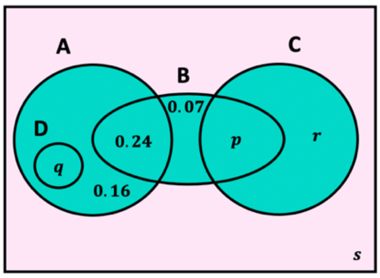
4 Diamond

4.1 With Set Notation

4.2 With Algebra

42) The Venn diagram shows 3 events, A, B and C, and their associated probabilities. Events B and C are mutually exclusive. Events A and C are independent. Showing your working, find the value of x, the value of y and the value of z.

43) The Venn diagram shows the probabilities associated with four events, *A*, *B*, *C* and *D*



i. Write down any pair of mutually exclusive events from A, B, C and D

Given that P(B) = 0.4

ii. find the value of p

Given also that A and B are independent

iii. find the value of q

Given further that P (B′ | C) = 0.64, find

iv. the value of r

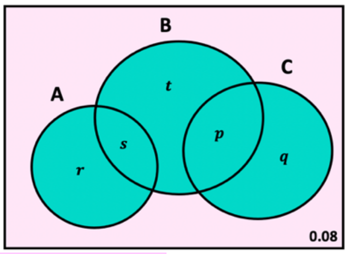
v. the value of s

44) Three events A, B and C are such that A and B are mutually exclusive and P(A)=0.2, P(C)=0.3 and P(A∪ $B) = 0.4$ and P(B∪ C)=0.34

i. Calculate P(B) and P(B∩ C)

ii. Determine whether B and C are independent

45) The Venn diagram shows three events, A,B and C where $p, q, r, s, and\ t$ are probabilities. P(A)=0.5, P(B)=0.6 and P(C)=0.25 and the events B and C are independent



i. Find the value of p and the value of q
ii. Find the value of r
iii. Hence write down the value of s and the value of t
iv. State, giving a reason, whether or not the events A and B are independent
v. Find P(B| A∪C)

46) P(A∪B)=0.7. P(A|B')=0.6. Find P(B)

47) A company has 1825 employees.
The employees are classified as professional, skilled or elementary.
The following table shows:
The number of employees in each classification
The two areas, A or B, where the employees live
An employee is chosen at random.

	A	B
Professional	740	380
Skilled	275	90
Elementary	260	80

Find the probability that this employee:

i. is skilled,

ii. lives in area B and is not a professional.

Some classifications of employees are more likely to work from home.

65% of professional employees in both area A and B work from home

40% of skilled employees in both area A and B work from home

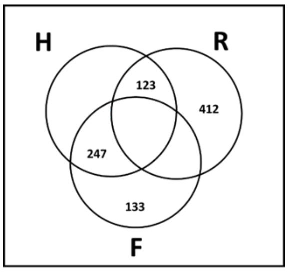
5% of elementary employees in both area A and B work from home

Event F is that the employee is a professional

Event H is that the employee works from home

Event R is that the employee is from area A.

iii. Using this information, complete this Venn diagram:



iv. Find $P(R' \cap F)$

v. Find $P((H \cup R)')$

vi. Find $P(F|H)$

48) Each of the 30 students in a class plays at least one of squash, hockey and tennis
18 students play squash
19 students play hockey
17 students play tennis
8 students play squash and hockey
9 students play hockey and tennis
11 students play squash and tennis

i. Find the number of students who play all three sports

A student is picked at random from the class

ii. Given that this student plays squash, find the probability that this student does not play hockey

Two different students are picked at random from the class, one after the other, without replacement.

iii. Given that the first student plays squash, find the probability that the second student plays hockey

49) The Venn diagram shows the numbers of students studying various subjects, in a year group of 100 students. A student is chosen at random from the 100 students. Then another student is chosen from the remaining students. Find the probability that the first student studies History and the second student studies Geography but not Psychology.

50) Given that the events, A, B and C are all independent and that P(A)$= x$, P(B)=y, P(C)$= z$, find in terms of x, y and z

i. P(A∩B∩C)
ii. P(A∪B∪C)
iii. P((A∪ B') ∩ C)

4.3 With Conditional - Bayes Theorem

51) Frances and George play tennis. When they play, the probability that Frances wins the first set of a match is $\frac{3}{5}$. If she wins the first set, the probability she wins the second set is $\frac{9}{10}$. If she loses the first set, the probability she wins the second set is $\frac{1}{2}$. Given that Francis wins the second set, find the probability that she also won the first set.

52) Only two international airlines fly daily into an airport. UN air has 70 flights a day and IS Air has 65 flights a day. Passengers flying with UN Air have an 18% probability of losing their luggage and passengers flying with IS Air have a 23% probability of losing their luggage. You overhear someone in the airport complain about her luggage being lost. Find the probability that she travelled with IS Air?

53) An influence virus is spreading through a city. A vaccination is available to protect against the virus. If a person has had the vaccination, the probability of catching the virus is 0.1; without the vaccination the probability is 0.3. The probability of a randomly selected person catching the virus is 0.22

i. Find the percentage of the population that has been vaccinated
ii. A randomly chosen person catches the virus. Find the probability that this person has been vaccinated?

54) 2 planes fly into Bristol airport, Easy Flights and Brian Air. Brian Air have three times are many flights into Bristol than Easy Flights. Brian Air lose passengers bags with a probability of $\frac{1}{6}$ and Easy Flights lose passenger bags with a probability of $\frac{1}{8}$. Given that a passenger is standing around complaining about having lost their bags, find the probability that the passenger has flown with Brian Air.

55) Computers are produced in two factories, A and B. A produces double the amount of computers than B. 15% of computers from factory A are faulty and 20% of computers from factory B are faulty. Given that a customer has a faulty computer, use Bayes' theorem to find the probability that it came from factory A.

56) A doctor is called to see a sick child. The doctor has prior information that 90% of sick children in that neighbourhood have the flu, whilst the other 10% are sick with measles. Let F stand for the event of a child being sick with the flu and M stand for an event of a child being sick with measles. Assume for simplicity $F \cup M = \Omega$ i.e. that no other maladies in that neighbourhood. A well-known symptom of measles is a rash (the event of having which we denote R). $P(R|M) = 0.95$. However, occasionally children with flu also develop rash, so that $P(R|F) = 0.08$. Upon examining the child, the doctor finds a rash. What is the probability that the child has measles?

57) In a study, physicians were asked what the odds of breast cancer would be in a woman who has initially thought to have 1% risk of cancer but who ended up with a positive mammogram result (a mammogram accurately classifies about 80% of cancerous tumours and 90% of benign tumours). 95 out of a hundred physicians estimated the probability of cancer to be about 75%. Do you agree?